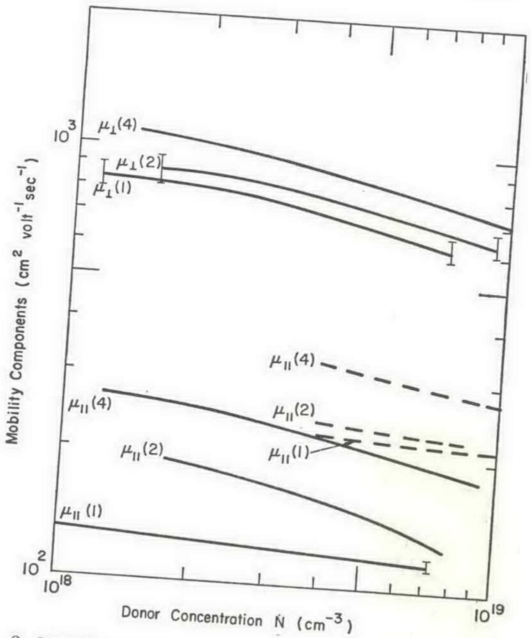


FIG. 8. Mobility components $\mu_{\parallel}$ and $\mu_\perp$ for As donors (full lines) and $\mu_{\parallel}$ for Sb donors (dashed lines) as a function of donor concentration at 1.2°K. The bracketed numbers indicate the number of lower conduction band valleys.

dominantly from the uncertainties in the transverse measurements. The mobility components of the four valley case were calculated from the zero-stress mobility. They are subject to the uncertainty of determining the value $K(4)=4.0$ discussed above. The dashed curves in Fig. 8 represent $\mu_{\parallel}$ for Sb-doped germanium for which $K$ was found to be $K=3.9\pm0.1$.

For both As and Sb doping the mobility increases with increasing number of lower valleys. This shows that the increase in mobility due to better screening outweighs the decrease of the mobility due to the lowering of the Fermi energy as the number of valleys is increased from 1 to 4.

Figure 8 demonstrates again the significantly larger scattering experienced by the electrons in the case of As-doping as compared to Sb-doping. This effect is stronger for $\mu_{\parallel}$ than for $\mu_\perp$, and it is larger for the 1 valley case than for the 4 valley case. This latter observation indicates that the principal cause for the different mobilities in Sb-doped and As-doped Ge cannot be intervalley scattering (caused by the central cell potential of the donors) of the kind discussed earlier[8] because in that case the mobilities of the one valley case should be nearly equal for the two donor elements.

It should be noted, however, that strong evidence for a considerable contribution of intervalley scattering in As-doped degenerate germanium was observed[9] in the electronic part of the ultrasonic attenuation at low temperatures. The ratio of intervalley to total scattering time was estimated to be about 10 for As and about 1000 for Sb donors independent of concentration in the range from $10^{18}$ to $3\times10^{19}$ cm$^{-3}$.

The size of the central impurity cell potential affects also the intravalley scattering rate. This effect has been discussed by Csavinszky.[21] He showed that the ratio $\mu(Sb)/\mu(As)$ is expected to be larger than unity and that this ratio increases with increasing impurity concentration because of changes in the screening of the impurity ions in qualitative agreement with the observations (see Fig. 8). A similar result was obtained by Tsidilkovski *et al.*[20] who calculated isotropic impurity scattering in the Born approximation using a scattering potential which was matched to the different individual donor ionization energies by a variational method.

It is clear from this evidence that the simple degenerate model, even if amended by intervalley scattering, is incapable of decreasing the piezoresistance effects in As-doped Ge. This conclusion is supported by the observation[12] of an anomalous negative contribution[22] to the magnetoresistance effect in this high concentration range. Furthermore, the behavior of interband tunneling in As-doped tunneling junctions shows that in contrast to the case where the donors are Sb the electrons cannot be considered associated with a particular valley.[23] To the donor wave functions, the large central cell potential of the As donors admixes Bloch wave contributions from the other valleys and from other regions of the Brillouin zone, particularly from the region around the zone center. As far as the electrons are shared by all four valleys and by the central minimum, they do not show the expected piezoresistance effects. The similar magnitude of $\Pi_{44}$ for As and Sb doping indicates that this admixture is relatively small. It depends, however, on the relative energies of the valleys and of the minimum at the zone center and hence it changes with stress. This can give rise to some of the observed changes of the piezoresistance beyond the calculated saturation stress.

Furthermore, some of the tail states may be sufficiently localized to cause a broad resonance scattering of the Breit-Wigner type as the stress moves these tail states past the continuum states near the Fermi level of the lower valley. This affects both the scattering probability and through a change of density of states at the Fermi level the screening of the scattering impurities. If the scattering to these tail states is stronger than intervalley scattering among four degenerate valleys then the larger ratio $\mu_{\parallel}(\mathrm{Sb})/\mu_{\parallel}(\mathrm{As})$ observed for the 1-valley case as compared to the 4-valley case can be understood.

## IV. SUMMARY AND CONCLUSIONS

The different effect of As and Sb donors on the transport properties of degenerate germanium becomes

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[21] P. Csavinszky, J. Phys. Soc. Japan, **16**, 1865 (1961).

[22] An anomalous positive contribution to the magnetoresistance found in *p*-type Ge seems to be closely related to the negative contribution found in As-doped Ge. See H. Roth, W. D. Straub, and W. Bernard, Phys. Rev. Letters **11**, 328 (1963).

[23] H. Fritzsche and J. J. Tiemann (to be published).

strongly apparent when the material is changed by stress to a 2-valley or a 1-valley semiconductor. In contrast to the case of Sb donors the mobilities of As-doped Ge have a simple power law dependence on concentration in the range $N > 10^{18}$ cm$^{-3}$. They cannot be described by a simple scattering model, however. The presence of an anomalous contribution to the magnetoresistance at these high As concentrations also shows the inadequacy of the simple degenerate model.

The mobility anisotropy factor in As-doped germanium was found to be $K(4)=4\pm0.4$, $K(2)=5\pm0.6$, and $K(1)=6\pm0.5$ for the 4-, 2-, and 1-valley case, respectively. The value $K(2)$ seems to increase and $K(1)$ to decrease slightly as $N$ increases from $10^{18}$ to $10^{19}$ cm$^{-3}$.

In contrast to the case of Sb donors the piezoresistance of degenerate As-doped germanium decreases beyond the theoretical saturation stress and approaches a constant value only at very high stresses. This indicates that the presence of localized tail states and their interaction with the valley or valleys shifted downwards by stress is more strongly pronounced in As-doped germanium.

The mobility ratio $\mu_{11}(\mathrm{Sb})/\mu_{11}(\mathrm{As})$ is found to be larger for the 1 valley case than for the 4-valley case. This indicates that simple intervalley scattering as discussed earlier[8] cannot be the primary reason for the lower mobility of As-doped germanium. It is possible, however, that the resonance scattering to the tail states of the valleys which are moved up by the stress is stronger than intervalley scattering among 4 degenerate valleys. Both of these scattering processes are expected to be less for Sb than for As donors because of the large difference in their central cell potentials.

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